

# Binary Matroids with Graphic Cocircuits

Konstantinos Papalamprou  
London School of Economics

Leonidas Pitsoulis  
Aristotle University of Thessaloniki

## Abstract

An excluded minor characterization for the class of binary signed-graphic matroids with graphic cocircuits is provided. In this report we present the necessary computations for the case analysis in the proof.

## An excluded minor characterization

The complete list of regular excluded minors for signed-graphic matroids is provided in [2], specifically:

**Theorem 1.** *A regular matroid  $M$  is signed-graphic if and only if  $M$  has no minor isomorphic to  $M^*(G_1), \dots, M^*(G_{29}), R_{15}$  or  $R_{16}$ .*

The matroids  $M^*(G_1), \dots, M^*(G_{29})$  are the cographic matroids of the 29 non-separable forbidden minors for projective planar graphs, while  $R_{15}$  and  $R_{16}$  are two special matroids whose binary compact representation matrices are given in [2] and in the next section of this Technical Report.

Clearly one could easily produce the complete list of binary excluded minors for signed-graphic matroids by adding to the list of the above 31 regular excluded minors the binary excluded minors for regular matroids (i.e.  $F_7$  and  $F_7^*$ ), since any binary signed-graphic matroid is also regular.

**Theorem 2.** *A binary matroid  $M$  is signed-graphic if and only if  $M$  has no minor isomorphic to  $M^*(G_1), \dots, M^*(G_{29}), R_{15}, R_{16}, F_7$  or  $F_7^*$ .*

We define a cocircuit  $Y$  of a matroid  $M$  to be *graphic* if the matroid  $M \setminus Y$  is graphic. The main result in this report is the complete list of excluded minors for the class of binary signed-graphic matroids with graphic cocircuits. Of importance for the proof of that result, Theorem 3 here, is the following Lemma.

**Lemma 1.** *If  $N$  is a minor of the matroid  $M$  then for any cocircuit  $C_N \in \mathcal{C}(N^*)$  there exists a cocircuit  $C_M \in \mathcal{C}(M^*)$  such that  $N \setminus C_N$  is a minor of  $M \setminus C_M$ .*

*Proof:* If  $N = M \setminus X / Y$  then by duality  $N^* = M / X \setminus Y$ . Therefore by the definitions of contraction and deletion of a set, we have that for any cocircuit  $C_N \in \mathcal{C}(N^*)$  there exists a cocircuit  $C_M \in \mathcal{C}(M^*)$  such that

- (i)  $C_N \subseteq C_M$ ,
- (ii)  $E(N) \cap C_M = C_N$ ,

which in turn imply that  $C_M - C_N \subseteq X$ . So we have

$$M \setminus C_M = M \setminus \{C_M - C_N\} \setminus C_N \succeq N \setminus C_N$$

□

**Theorem 3.** *Let  $M$  be a binary matroid such that all its cocircuits are graphic. Then,  $M$  is signed-graphic if and only if  $M$  has no minor isomorphic to  $M^*(G_{17})$ ,  $M^*(G_{19})$ ,  $F_7$  or  $F_7^*$ .*

*Proof:*  $M$  must contain a minor isomorphic to some matroid in the set

$$\mathcal{M} = \{M^*(G_1), \dots, M^*(G_{16}), M^*(G_{18}), M^*(G_{20}), \dots, M^*(G_{29}), R_{15}^*, R_{16}^*\}.$$

By case analysis, verified also by the MACEK software [1], it can be shown that for each matroid  $N \in \mathcal{M}$  there exists a cocircuit  $Y_N \in \mathcal{C}(N^*)$  such that the matroid  $N \setminus Y_N$  contains an  $M^*(K_{3,3})$  or an  $M^*(K_5)$  as a minor, which implies that  $N \setminus Y_N$  is not graphic. Therefore, by Lemma 1, there is a cocircuit  $Y_M \in \mathcal{C}(M^*)$  such that  $N \setminus Y_N$  is a minor of  $M \setminus Y_M$ . Thus,  $M \setminus Y_M$  is not graphic which is in contradiction with our assumption that  $M$  has graphic cocircuits.  $\square$

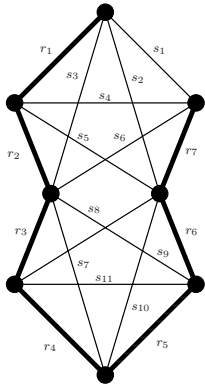
As already mentioned, this Technical Report is mainly devoted to the computations performed using the MACEK software [1] appearing in the proof of Theorem 3. These computations are provided in detail in the next section.

## MACEK computations

Each case, i.e. matroid in  $\mathcal{M}$  in the proof of Theorem 3, will be examined separately and specifically:

- for each cographic matroids in  $\mathcal{M}$ , a compact representation matrix of its dual graphic matroid along with the associated graph  $((G_1, \dots, G_{16}, G_{18}, G_{20}, \dots, G_{29}))$  are provided. It is clear that due to matroid duality, it is enough to find a circuit  $C$  in each  $M \in \{M(G_1), \dots, M(G_{16}), M(G_{18}), M(G_{20}), \dots, M(G_{29})\}$  such that  $M/C$  contains an  $M(K_{3,3})$ - or an  $M(K_5)$ -minor. The advantage of working with the duals of the cographic matroids in  $\mathcal{M}$  is that someone could graphically see that by contracting a cycle (i.e. the one corresponding to  $C$ ) in the associated graph, a minor isomorphic to  $K_{3,3}$  or  $K_5$  is contained in the resulting graph. Therefore, in that case, the MACEK computations may be seen just as a validation tool.
- for each of the two non-cographic matroid in  $\mathcal{M}$  (i.e.  $R_{15}$  and  $R_{16}$ ), a compact representation matrix is provided along with the cocircuit to be deleted. The MACEK commandss and the outputs showing that each of the resulting matroids contains an  $M^*(K_{3,3})$  or an  $M^*(K_5)$  as a minor are given.

**The matroid  $M(G_1)$ :**



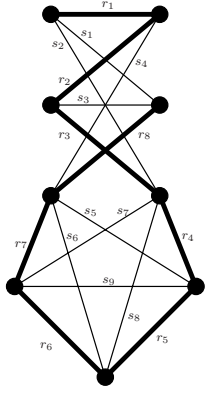
$$g_1 = \begin{matrix} & \begin{matrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_1)/\{r_1, s_1, s_3\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract -1;!contract -3;!minor' g1 '{grK5,grK33}'`

**Output:** The #1 matroid `[g1~c1~c-1~c-3]` +HAS+ minor #1 `[grK5]` in the list `{grK5 grK33}`.

The matroid  $M(G_2)$ :



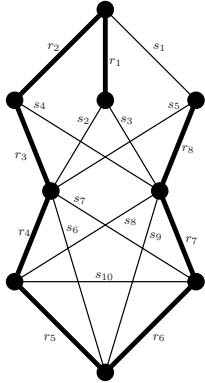
$$g_2 = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_2)/\{r_4, r_5, s_9\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 4;!contract 5;!contract -9;!minor' g2 '{grK5,grK33}'`

**Output:** The #1 matroid `[g2~c4~c5~c-9]` +HAS+ minor #2 `[grK33]` in the list `{grK5 grK33}`.

The matroid  $M(G_3)$ :

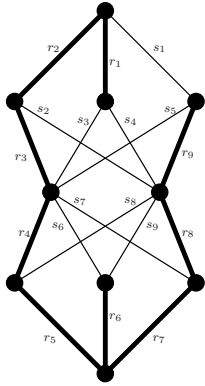


$$g_3 = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_3)/\{r_1, r_2, r_3, s_2\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract -2;!minor' g3 '{grK5,grK33}'`

**Output:** The #1 matroid `[g3~c1~c2~c3~c-2]` +HAS+ minor #1 `[grK5]` in the list `{grK5 grK33}`.



$$g_4 = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

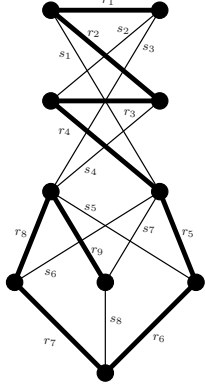
**The matroid  $M(G_4)$ :**

$M(G_4)/\{r_4, r_5, r_6, s_6\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 4;!contract 5;!contract 6;!contract -6;!minor' g4 '{grK5,grK33}'`

**Output:** The #1 matroid [g4~c4~c5~c6~c-6] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

The matroid  $M(G_5)$ :



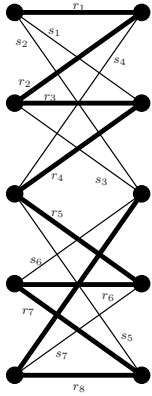
$$g_5 = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

$M(G_5)/\{r_7, r_8, r_9, s_8\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** ./macek -pGF2 '!contract 7;!contract 8;!contract 9;!contract -8;!minor' g5 '{grK5,grK33}'

**Output:** The #1 matroid [g5~c7~c8~c9~c-8] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

The matroid  $M(G_6)$ :



$$g_6 = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$M(G_6)/\{r_6, r_7, r_8, s_7\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** ./macek -pGF2 '!contract 6;!contract 7;!contract 8;!contract -7;!minor' g6 '{grK5,grK33}'

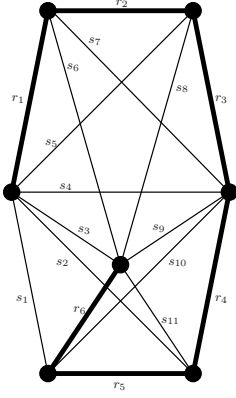
**Output:** The #1 matroid [g6~c6~c7~c8~c-7] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

The matroid  $M(G_7)$ :

$M(G_7)/\{r_1, r_2, s_5\}$  contains an  $M(K_5)$ -minor.

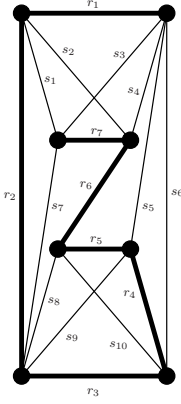
**Command:** ./macek -pGF2 '!contract 1;!contract 2;!contract -5;!minor' g7 '{grK5,grK33}'

**Output:** The #1 matroid [g7~c1~c2~c-5] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.



$$g_7 = \begin{array}{c} \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{matrix} \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

The matroid  $M(G_8)$ :



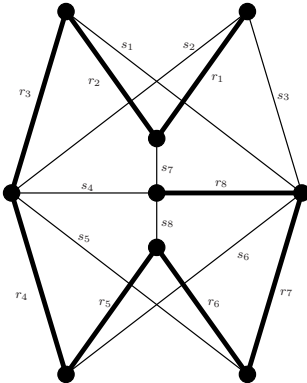
$$g_8 = \begin{array}{c} \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{matrix} \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$M(G_8)/\{r_4, s_5, s_6\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 4;!contract -5;!contract -6;!minor' g8 '{grK5,grK33}'`

**Output:** The #1 matroid [g8~c4~c-5~c-6] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

The matroid  $M(G_9)$ :



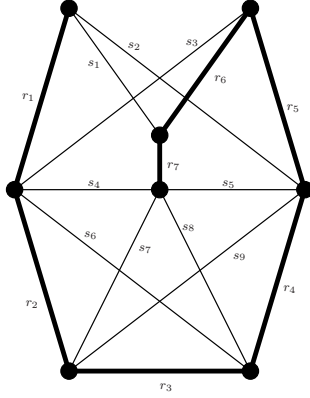
$$g_9 = \begin{array}{c} \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$M(G_9)/\{r_1, r_2, r_3, s_2\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract -2;!minor' g9 '{grK5,grK33}'`

**Output:** The #1 matroid [g9~c1~c2~c3~c-2] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

**The matroid  $M(G_{10})$ :**



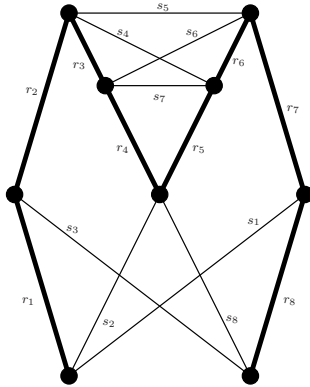
$$g_{10} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{10})/\{r_1, r_6, s_1, s_3\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 6;!contract -1;!contract -3;!minor' g10 '{grK5,grK33}'`

**Output:** The #1 matroid [g10~c1~c6~c-1~c-3] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

**The matroid  $M(G_{11})$ :**



$$g_{11} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$M(G_{11})/\{r_3, r_6, s_5, s_7\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 3;!contract 6;!contract -5;!contract -7;!minor' g11 '{grK5,grK33}'`

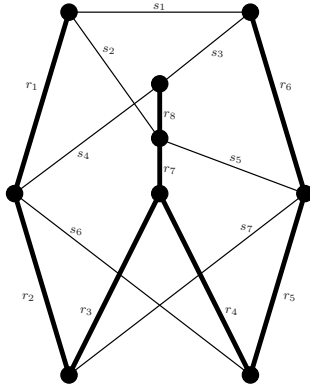
**Output:** The #1 matroid [g11~c3~c6~c-5~c-7] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

**The matroid  $M(G_{12})$ :**

$M(G_{12})/\{r_8, s_1, s_2, s_3\}$  contains an  $M(K_{3,3})$ -minor.

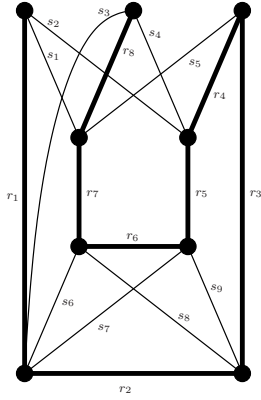
**Command:** `./macek -pGF2 '!contract 8;!contract -1;!contract -2;!contract -3;!minor' g12 '{grK5,grK33}'`

**Output:** The #1 matroid [g12~c8~c-1~c-2~c-3] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.



$$g_{12} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

**The matroid  $M(G_{13})$ :**



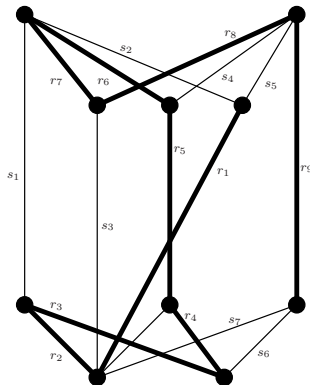
$$g_{13} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$M(G_{13})/\{r_2, r_6, s_6, s_9\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 2;!contract 6;!contract -6;!contract -9;!minor' g13 '{grK5,grK33}'`

**Output:** The #1 matroid [g13~c2~c6~c-6~c-9] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

**The matroid  $M(G_{14})$ :**



$$g_{14} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

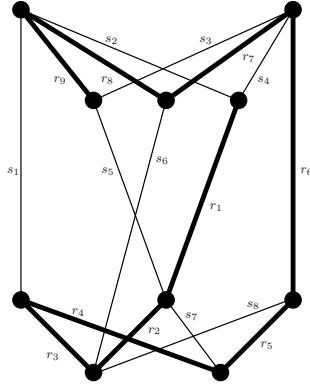
$M(G_{14})/\{r_7, r_8, s_2, s_5\}$  contains an  $M(K_{3,3})$ -minor.



**Command:** `./macek -pGF2 '!contract 7;!contract 8;!contract -2;!contract -5;!minor' g14 '{grK5,grK33}'`

**Output:** The #1 matroid  $[g14 \sim c7 \sim c8 \sim c-2 \sim c-5]$  +HAS+ minor #2  $[grK33]$  in the list  $\{grK5 \text{ } grK33\}$ .

**The matroid  $M(G_{15})$ :**



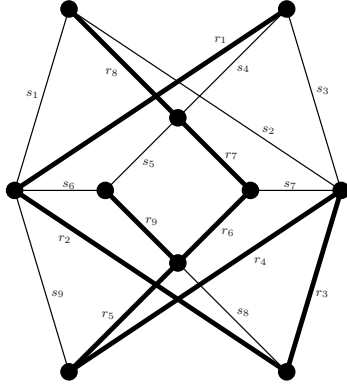
$$g_{15} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{15})/\{r_7, r_8, r_9, s_3\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 7;!contract 8;!contract 9;!contract -3;!minor' g15 '{grK5,grK33}'`

**Output:** The #1 matroid  $[g15 \sim c7 \sim c8 \sim c9 \sim c-3]$  +HAS+ minor #2  $[grK33]$  in the list  $\{grK5 \text{ } grK33\}$ .

**The matroid  $M(G_{16})$ :**



$$g_{16} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{16})/\{r_6, r_7, r_9, s_5\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 6;!contract 7;!contract 9;!contract -5;!minor' g16 '{grK5,grK33}'`

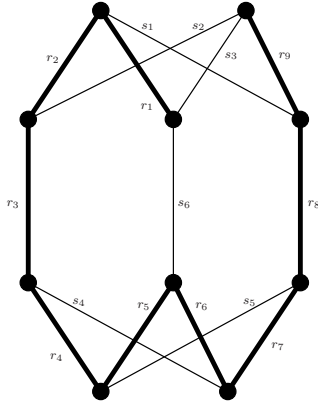
**Output:** The #1 matroid  $[g16 \sim c6 \sim c7 \sim c9 \sim c-5]$  +HAS+ minor #2  $[grK33]$  in the list  $\{grK5 \text{ } grK33\}$ .

**The matroid  $M(G_{18})$ :**

$M(G_{18})/\{r_4, r_7, s_4, s_5\}$  contains an  $M(K_{3,3})$ -minor.

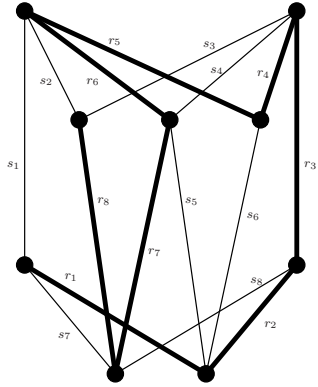
**Command:** `./macek -pGF2 '!contract 4;!contract 7;!contract -4;!contract -5;!minor' g18 '{grK5,grK33}'`

**Output:** The #1 matroid  $[g18 \sim c4 \sim c7 \sim c-4 \sim c-5]$  +HAS+ minor #2  $[grK33]$  in the list  $\{grK5 \text{ } grK33\}$ .



$$g_{18} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

**The matroid  $M(G_{20})$ :**



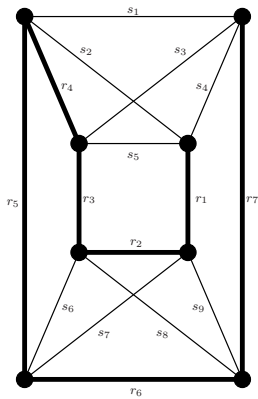
$$g_{20} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{20})/\{r_1, r_2, s_7, s_8\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 2;!contract -7;!contract -8;!minor' g20 '{grK5,grK33}'`

**Output:** The #1 matroid [g20~c1~c2~c-7~c-8] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

**The matroid  $M(G_{21})$ :**



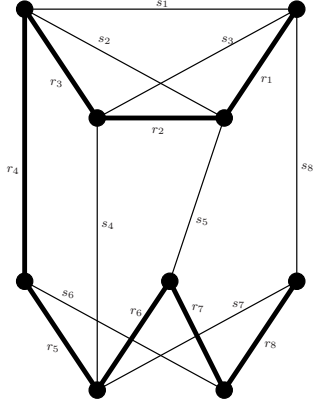
$$g_{21} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{21})/\{r_4, s_2, s_5\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 4;!contract -2;!contract -5;!minor' g21 '{grK5,grK33}'`

**Output:** The #1 matroid [g21~c4~c-2~c-5] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

**The matroid  $M(G_{22})$ :**



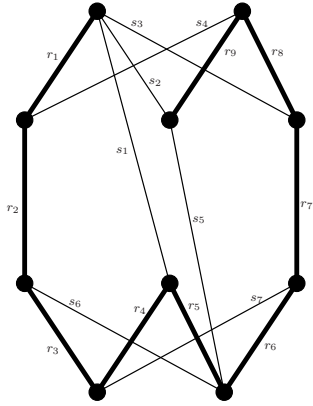
$$g_{22} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

$M(G_{22})/\{r_5, r_6, r_7, s_6\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 5;!contract 6;!contract 7;!contract -6;!minor' g22 '{grK5,grK33}'`

**Output:** The #1 matroid [g22~c5~c6~c7~c-6] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

**The matroid  $M(G_{23})$ :**

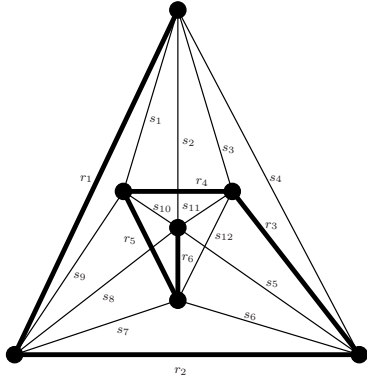


$$g_{23} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{23})/\{r_1, r_8, s_3, s_4\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 8;!contract -3;!contract -4;!minor' g23 '{grK5,grK33}'`

**Output:** The #1 matroid [g23~c1~c8~c-3~c-4] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.



$$g_{24} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} & s_{12} \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

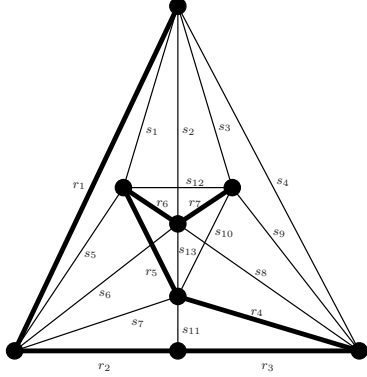
**The matroid  $M(G_{24})$ :**

$M(G_{24})/\{r_1, s_1, s_9\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract -1;!contract -9;!minor' g24 '{grK5,grK33}'`

**Output:** The #1 matroid [g24~c1~c-1~c-9] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

**The matroid  $M(G_{25})$ :**



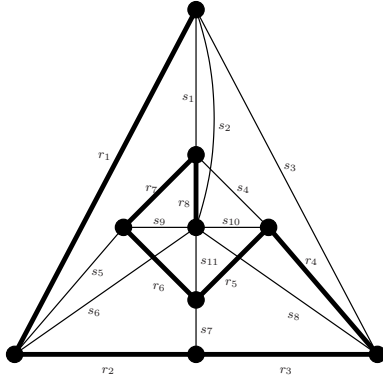
$$g_{25} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} & s_{12} & s_{13} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$M(G_{25})/\{r_5, s_{10}, s_{12}\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 5;!contract -10;!contract -12;!minor' g25 '{grK5,grK33}'`

**Output:** The #1 matroid [g25~c5~c-10~c-12] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

**The matroid  $M(G_{26})$ :**



$$g_{26} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$$

$M(G_{26})/\{r_5, r_6, r_7, s_4\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 5;!contract 6;!contract 7;!contract -4;!minor' g26 '{grK5,grK33}'`

**Output:** TThe #1 matroid [g26~c5~c6~c7~c-4] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

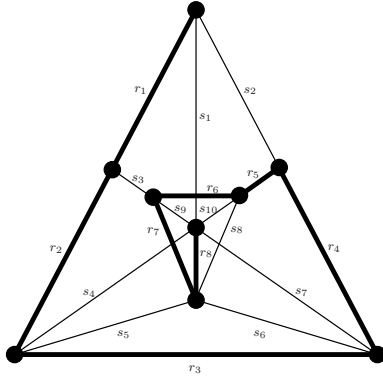
**The matroid  $M(G_{27})$ :**

$M(G_{27})/\{r_1, r_2, r_3, r_4, s_2\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract 4;!contract -2;!minor' g27`

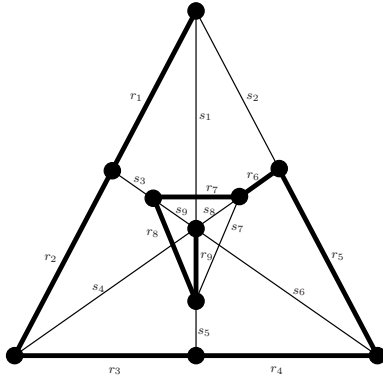
`'{grK5,grK33}'`

**Output:** The #1 matroid [g27~c1~c2~c3~c4~c-2] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.



$$g_{27} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

The matroid  $M(G_{28})$ :



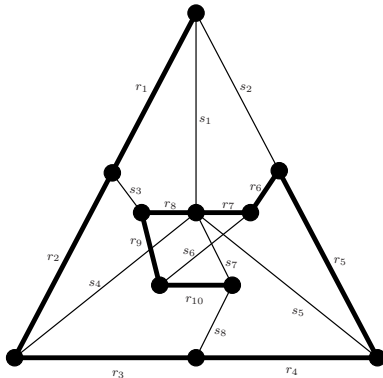
$$g_{28} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$M(G_{28})/\{r_1, r_2, r_3, r_4, r_5, s_2\}$  contains an  $M(K_5)$ -minor.

**Command:** `./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract 4;!contract 5;!contract -2;!minor'`  
`g28 '{grK5,grK33}'`

**Output:** The #1 matroid [g28~c1~c2~c3~c4~c5~c] +HAS+ minor #1 [grK5] in the list {grK5 grK33}.

The matroid  $M(G_{29})$ :



$$g_{29} = \begin{array}{c} \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \\ r_9 \\ r_{10} \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$M(G_{29})/\{r_1, r_2, r_3, r_4, r_5, s_2\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** ./macek -pGF2 '!contract 1;!contract 2;!contract 3;!contract 4;!contract 5;!contract -2;!minor' g29  
'{grK5,grK33}'

**Output:** The #1 matroid [g5~c7~c8~c9~c-8] +HAS+ minor #2 [grK33] in the list {grK5 grK33}.

**The matroid  $R_{15}$ :**

$$r_{15} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$R_{15} \setminus \{r_6, r_7, s_8\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** ./macek -pGF2 '!delete 6;!delete 7;!delete -8;!minor' r15 '{"grK5;!dual","grK33;!dual"}'

**Output:** The #1 matroid [r15~d6~d7~d-8] +HAS+ minor #2 [grK33#] in the list {grK5# grK33#}.

**The matroid  $R_{16}$ :**

$$r_{16} = \begin{matrix} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ \begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$M(R_{16}) \setminus \{r_8, s_1, s_3, s_8\}$  contains an  $M(K_{3,3})$ -minor.

**Command:** ./macek -pGF2 '!delete 8;!delete -1;!delete -3;!delete -8;!minor' r16  
'{"grK5;!dual","grK33;!dual"}'

**Output:** The #1 matroid [r16~d8~d-1~d-3~d-8] +HAS+ minor #2 [grK33#] in the list {grK5# grK33#}.

## Acknowledgements

This research has been funded by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework (NSRF) - Research Funding Program: Thalys. Investing in knowledge society through the European Social Fund.

## References

- [1] P. Hliněný. MACEK 1.2+ MATroids Computed Efficiently Kit, 2007.  
<http://www.fi.muni.cz/~hlineny/MACEK/>.
- [2] H. Qin and D. Slilaty and X. Zhou. The regular excluded minors for signed-graphic matroids. *Combinatorics, Probability and Computing*, 18:953–978, 2009.



